

Multiple Assets: Two-Fund Separation

I. Mean-Variance Utility

Remember from last class that an investor with CARA utility

- i. Invest more in risky asset when expected return of the risky asset is higher
- ii. Invest less in risky asset when variance of the risky asset is higher

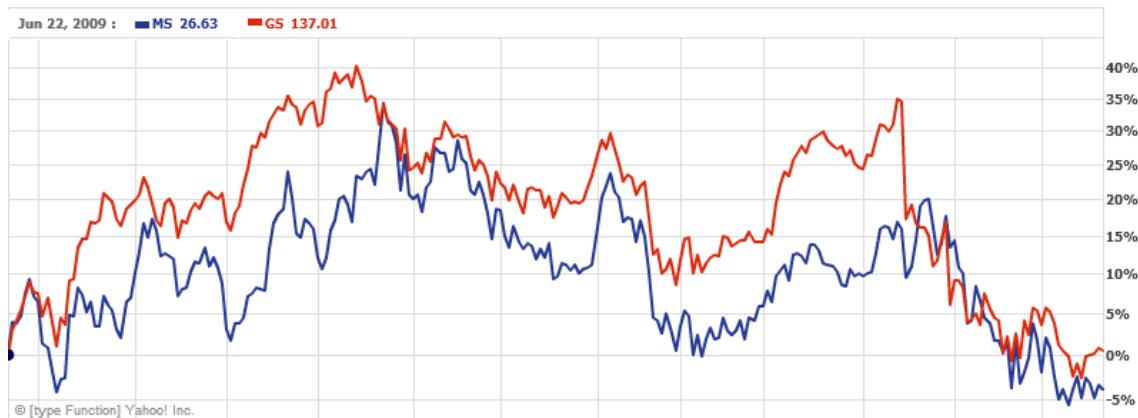
The investor likes higher expected return but hates higher variance. Furthermore, *these are the only two things she cares about in her investment decision.*

Under suitable distributional assumptions, this property holds for all three utility functions in the HARA family mentioned previously.

II. Multiple Assets: Doing the maximization again?

For independent normal distributions and CARA utility, we could repeat the maximization exercise in last class. We can do this because the sum of independent normal distributions is still normal.

Unfortunately, assets are rarely completely independent in the real world. Furthermore, there are cases where we are specifically interested in correlated assets.



Concept: *Covariance*

$$Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$$

Where

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])]$$

Imagine even having ten assets.

III. Multiple Assets: The more realistic way

For mean-variance utilities, there is a much easier way to do things. Since the investor only cares about the mean and the variance of her risky investment, we can first find all the combination of investments that give the highest mean for any given variance.

Consider the following example,

Stock	P = .5	P = .5
A	4	2
B	2	4
C	2	0

Let's calculate the variance for each stock and put them on a graph.

Any reason to buy Stock C?

What about a *mutual fund* with $\frac{1}{2}$ Stock A and $\frac{1}{2}$ stock B?

In general, how would the graph look like if we put in all possible combination of assets?

Without a risk-free asset, the upper portion of the line formed is the efficient in terms of the tradeoff between return and risk. Thus it is called the *efficient frontier*.

Now add in the risk-free return, with free lending and borrowing.

The efficient frontier is now the line extending from the risk-free return, and which tangents to the curve that joins all efficient risky asset. This is called the *capital allocation line (CAL)*.